# 2025 CHICAGO AREA ALL-STAR MATH TEAM TRYOUTS

**Problems 1–2** **Name** ____________________________

**Time Limit: 10 minutes** **School** ____________________________

**1.** Compute the number of two-digit positive integers $k$ for which $4k + 1$ is a multiple of 9.

**2.** Let $f(x) = 4x - x^2$. Define $f_n(x)$ to be the result of recursively applying $f$ to $x$ $n$ times. That is, $f_2(x) = f(f(x))$, $f_3(x) = f(f(f(x)))$, and so forth.

Compute $f_{2024}(2) + f_{2025}(2 + \sqrt{3}) + f_{2026}(2 - \sqrt{3})$.

**ANSWER TO PROBLEM 1**

**ANSWER TO PROBLEM 2**

# 2025 CHICAGO AREA ALL-STAR MATH TEAM TRYOUTS

**Problems 3–4** **Name** ____________________________

**Time Limit: 10 minutes** **School** ____________________________

**3.** Determine all ordered pairs $(m, n)$ such that a convex $m$-gon has 19 more diagonals than a convex $n$-gon.

**4.** Compute the number of divisors of 12! that have exactly one trailing zero (that is, the units digit is zero, but the next digit is not).

**ANSWER TO PROBLEM 3**

**ANSWER TO PROBLEM 4**

# 2025 CHICAGO AREA ALL-STAR MATH TEAM TRYOUTS

**Problems 5–6** **Name** ____________________

**Time Limit: 10 minutes** **School** ____________________

**5.** Consider the points $A(0,0)$, $B(14,0)$, and $X(5,12)$. Compute the sum of the $x$-coordinates of all points that lie on the $x$-axis and are equidistant to the two lines $\overleftrightarrow{AX}$ and $\overleftrightarrow{BX}$.

**6.** Griffin has a metronome (a device musicians use to help keep an even rhythm) that he will set to some integer between 40 and 208 (inclusive), which is the number of beats per minute (times per minute that the unit produces a beat sound to keep time). He will also set the metronome to tick 1, 2, 3, or 4 times per beat. Compute the number of distinct numbers of ticks per minute that this metronome will produce with these combinations of settings.

**ANSWER TO PROBLEM 5**

**ANSWER TO PROBLEM 6**

## 2025 CHICAGO AREA ALL-STAR MATH TEAM TRYOUTS

**Problems 7–8** **Name** ____________________________

**Time Limit: 10 minutes** **School** ____________________________

**7.** The graph of the relation $4x^3 + 2x - y^5 + 12 = 0$ is first translated 4 units to the right, then dilated horizontally by a factor of $1/3$. It is then reflected over the $x$-axis and, finally, translated 1 unit down. The resulting graph has an equation of the form $g(x, y) = 0$ where $g$ is a polynomial in two variables with integer coefficients that have no common factor. Compute the absolute value of the sum of the coefficients of $g$.

**8.** Everett plays a trivia game each day to earn treats. Each time he answers a question correctly he earns one treat and goes on to try to answer another question. If he answers a question incorrectly he does not earn a treat for that question and may not play the game again until the next day. Everett can answer any question correctly with probability $p$, independent of any other considerations. Everett expects to earn 2.5 treats per day. Compute $p$.

**ANSWER TO PROBLEM 7**

**ANSWER TO PROBLEM 8**

# 2025 CHICAGO AREA ALL-STAR MATH TEAM TRYOUTS

**Problems 9–10** **Name** ____________________________

**Time Limit: 10 minutes** **School** ____________________________

**9.** Jack and Chris are watching shooting stars. Shooting stars appear at random, uniformly distributed in time. Within any 45-minute interval, they have a 50% chance of seeing a shooting star. Compute the probability that on average, they see a shooting star within the next hour.

**10.** Define a sequence as follows: $a_1 = 20$, and for $n > 1$, $a_n$ is the number of times that $a_{n-1}$ has occurred in the sequence. Thus, the sequence starts $20, 1, 1, 2, 1, 3, \ldots$. Compute the least $k$ for which $a_k = 25$.

**ANSWER TO PROBLEM 9**

**ANSWER TO PROBLEM 10**

# 2025 CHICAGO AREA ALL-STAR MATH TEAM TRYOUTS

**Problems 11–12** **Name** ____________________________

**Time Limit: 10 minutes** **School** ____________________________

**11.** Let $S$ be the set of points in the plane that are three times as far from $(1,0)$ as they are from $(-1,0)$. Compute the area of the region bounded by $S$.

**12.** Compute the value of

$$\sum_{s \in S} \sum_{t \in T} \frac{1}{s+t}$$

where $S$ is the set of complex roots of $x^2 = 2$ and $T$ is the set of complex roots of $x^3 = 3$.

**ANSWER TO PROBLEM 11**

**ANSWER TO PROBLEM 12**

**Problems 13–14** **Name** ____________________________

**Time Limit: 10 minutes** **School** ____________________________

**13.** Point $P$ lies outside circle $O$. Ray $\overrightarrow{PO}$ intersects the circle at points $A$ and $B$, with $A$ between $P$ and $B$. Another ray with vertex $P$ intersects circle $O$ at points $C$ and $D$, with $C$ between $P$ and $D$. Given that $\angle DPB \cong \angle DOC$, $PA = 3$, and $PB = 7$, compute $PD$.

**14.** A car drives for two kilometers, turns $60^\circ$ to the right (this means that the angle between the “straight ahead” ray and the “direction the car is facing now ray is $60^\circ$)”, drives for three more kilometers, turns another $60^\circ$ to the right, and drives for five more kilometers. Compute the number kilometers the car is from its starting point.

**ANSWER TO PROBLEM 13**

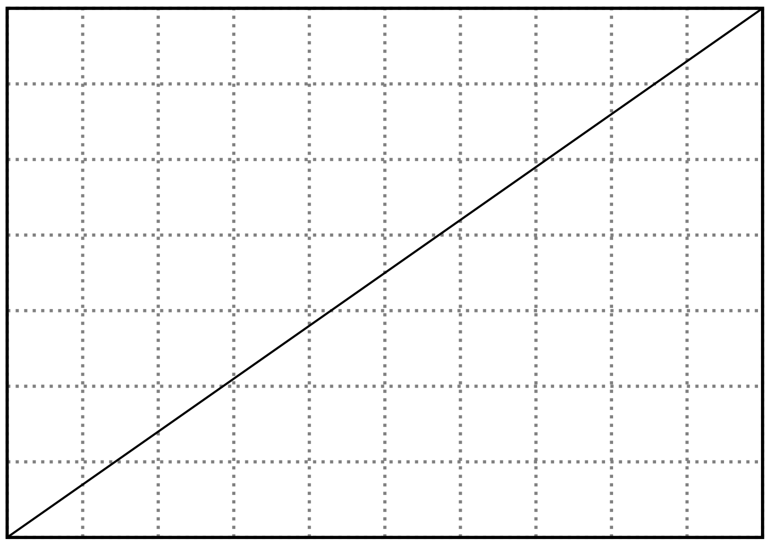
**ANSWER TO PROBLEM 14**

**Problems 15–16** **Name** ____________________________

**Time Limit: 10 minutes** **School** ____________________________

**15.** Compute the least three-digit positive integer such that the sum of its digits in base nine, the sum of its digits in base ten, and the sum of its digits in base eleven are all equal.

**16.** Consider a $10 \times 7$ rectangle subdivided into $1 \times 1$ squares. A diagonal of the rectangle is drawn, cutting some of the squares into triangles, pentagons, and trapezoids. Compute the total area of all the pentagons.



**ANSWER TO PROBLEM 15**

**ANSWER TO PROBLEM 16**

## 2025 CHICAGO AREA ALL-STAR MATH TEAM TRYOUTS

**Problems 17–18** **Name** ____________________________

**Time Limit: 10 minutes** **School** ____________________________

**17.** The four-digit number 8712 is four times its reverse. That is, $8712 = 4 \cdot 2178$. Compute the greatest positive nine-digit number that is nine times its reverse (please only consider numbers written in base 10).

**18.** Complete the cross-number puzzle below, where each Across answer is a 4-digit number and each Down answer is a 3-digit number. No answer begins with the digit 0. **Your answer must be written in the grid at the bottom of the page; the grid to the right is only for scratch work!**

| Across | Down |
|---|---|
| 1. A palindrome | 1. A perfect cube |
| 5. A permutation of the digits of 2025 | 2. A perfect square |
| | 3. A perfect square |
| 6. A multiple of 50 | 4. A perfect square |

| | | | |
|---|---|---|---|
| 1 | 2 | 3 | 4 |
| 5 | | | |
| 6 | | | |

**ANSWER TO PROBLEM 17**

**ANSWER TO PROBLEM 18**

| | | | |
|---|---|---|---|
| 1 | 2 | 3 | 4 |
| 5 | | | |
| 6 | | | |

## 2025 CHICAGO AREA ALL-STAR MATH TEAM TRYOUTS

**Problems 19–20** **Name** ______________________________

**Time Limit: 10 minutes** **School** ______________________________

**19.** Let $P(n)$ represent the probability that three distinct vertices chosen randomly from a regular $n$-gon are also the vertices of a right triangle. Compute the sum of the values of $n$ for which $\frac{1}{5} \leq P(n) \leq \frac{1}{3}$.

**20.** Let $a$ be the numerator when the expression

$$\frac{1}{1} + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \frac{1}{5} + \frac{1}{6} + \frac{1}{7} + \frac{1}{8} + \frac{1}{9} + \frac{1}{10}$$

is written as a single fraction in simplest form. Compute the remainder when $a$ is divided by 11.

**ANSWER TO PROBLEM 19**

**ANSWER TO PROBLEM 20**

**Problems 21–22** **Name** ____________________________

**Time Limit: 10 minutes** **School** ____________________________

**21.** In the OkayMon trading card game there are 6 different types of creatures. The cards are sold in packs, and each pack has exactly 3 different types of creatures in it. In any given pack, all possible combinations of 3 different creature types are equally likely.

Fritz purchase three packs of cards. Compute the probability that each of the six different types of creatures appear at least once within those three packs.

**22.** Froggy hops by jumping exactly one meter in a random direction. The direction chosen for each hop is independent from the direction chosen for each other hop. Compute the probability that Froggy is within one meter of his starting point after three hops.

**ANSWER TO PROBLEM 21**

**ANSWER TO PROBLEM 22**

## 2025 CHICAGO AREA ALL-STAR MATH TEAM TRYOUTS

**Problems 23–24** **Name** ______________________________

**Time Limit: 10 minutes** **School** ______________________________

**23.** In $\triangle ABC$, point $E$ is the midpoint of $\overline{AC}$. Point $D$ is located on $\overline{AB}$ so that $AD : DB = 1 : 2$. Cevians $\overline{CD}$ and $\overline{BE}$ intersect at $F$ and are perpendicular, and $CF : FB = 3 : 4$. Compute $\cos(A)$.

**24.** Compute the number of positive rational numbers $b$ for which the function

$$f(x) = \log_b(x + \sqrt{x^2 - 64}) + \log_b(x - \sqrt{x^2 - 64})$$

contains an integer in its range.

**ANSWER TO PROBLEM 23**

**ANSWER TO PROBLEM 24**

**2025 CHICAGO AREA ALL-STAR MATH TEAM TRYOUTS**

**Part I Answers**

| | | | |
|---|---|---|---|
| 1. | 10 | 2. | 6 |
| 3. | $(21, 20)$ and $(12, 10)$ | 4. | 264 |
| 5. | $-169/2$ (or equivalent) | 6. | 463 |
| 7. | 38 | 8. | $5/7$ |
| 9. | $1 - 2^{-4/3}$ (or equivalent) | 10. | 562 |
| 11. | $9\pi/16$ | 12. | 36 |
| 13. | 5 | 14. | 7 |

**2025 CHICAGO AREA ALL-STAR MATH TEAM TRYOUTS**

**Part II Answers**

15. 370

16. 107/10 (or equivalent)

17. 989,999,901

18. $\begin{array}{|c|c|c|c|} \hline 1 & 2 & 2 & 1 \\ \hline 2 & 5 & 2 & 0 \\ \hline 5 & 6 & 5 & 0 \\ \hline \end{array}$

19. 52

20. 0

21. 147/400

22. 1/4 (or equivalent)

23. $\sqrt{2}/2$

24. 8

**Tryout Results 2025**

Item Analysis

($n = xxx$ for Q1–Q24)

| Q | # right | % |
|---|---|---|
| 1 | | % |
| 2 | | % |
| 3 | | % |
| 4 | | % |
| 5 | | % |
| 6 | | % |
| 7 | | % |
| 8 | | % |
| 9 | | % |
| 10 | | % |
| 11 | | % |
| 12 | | % |

| Q | # right | % |
|---|---|---|
| 13 | | % |
| 14 | | % |
| 15 | | % |
| 16 | | % |
| 17 | | % |
| 18 | | % |
| 19 | | % |
| 20 | | % |
| 21 | | % |
| 22 | | % |
| 23 | | % |
| 24 | | % |

Distribution of Top Scores

| Score | $n$ |
|---|---|
| 24 | |
| 23 | |
| 22 | |
| 21 | |
| 20 | |
| 19 | |
| 18 | |
| 17 | |
| 16 | |
| 15 | |
| 14 | |